Blind zkSNARKs

for Private Proof Delegation and Verifiable Computation over Encrypted Data

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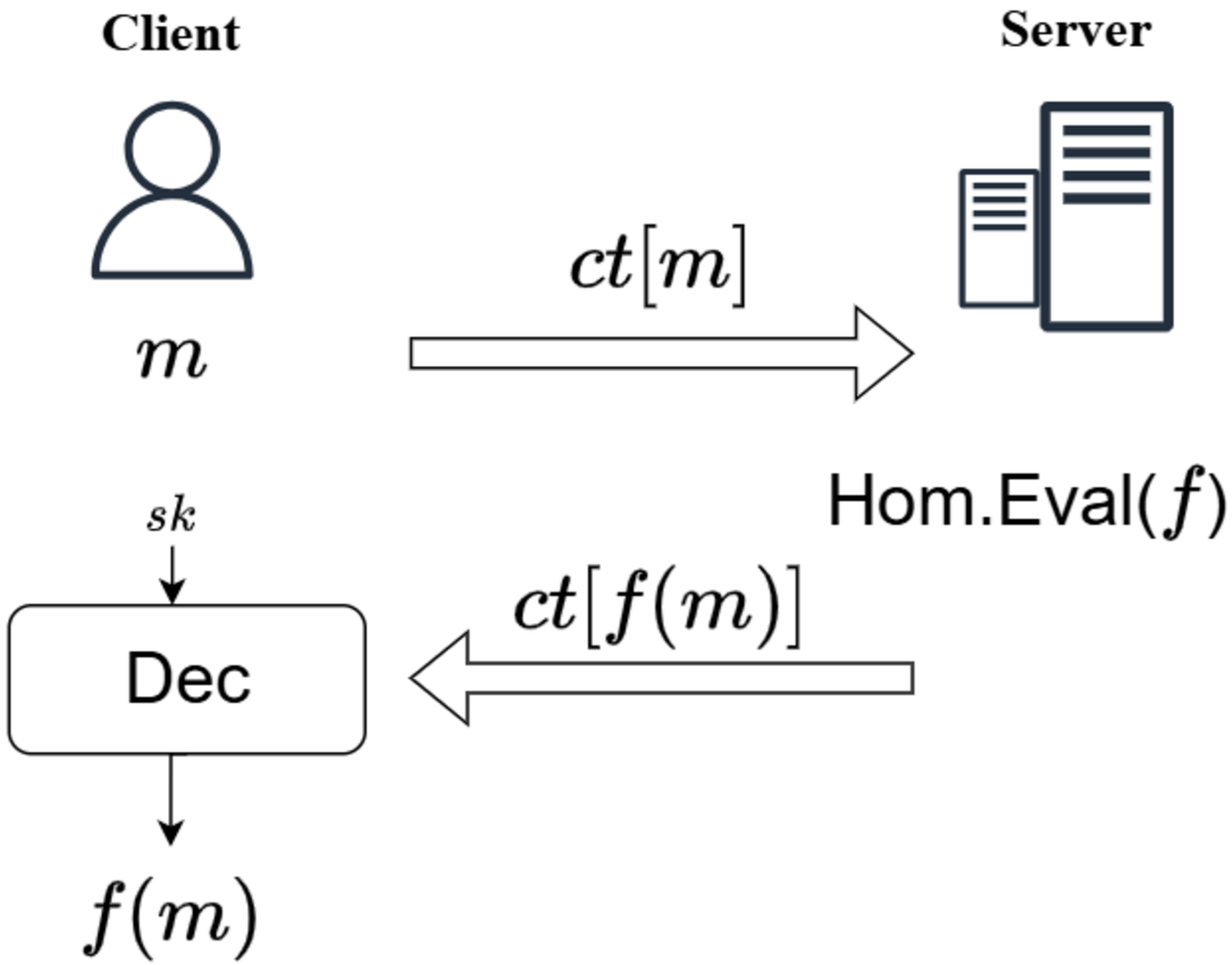
FHE enables computation over encrypted data (COED)



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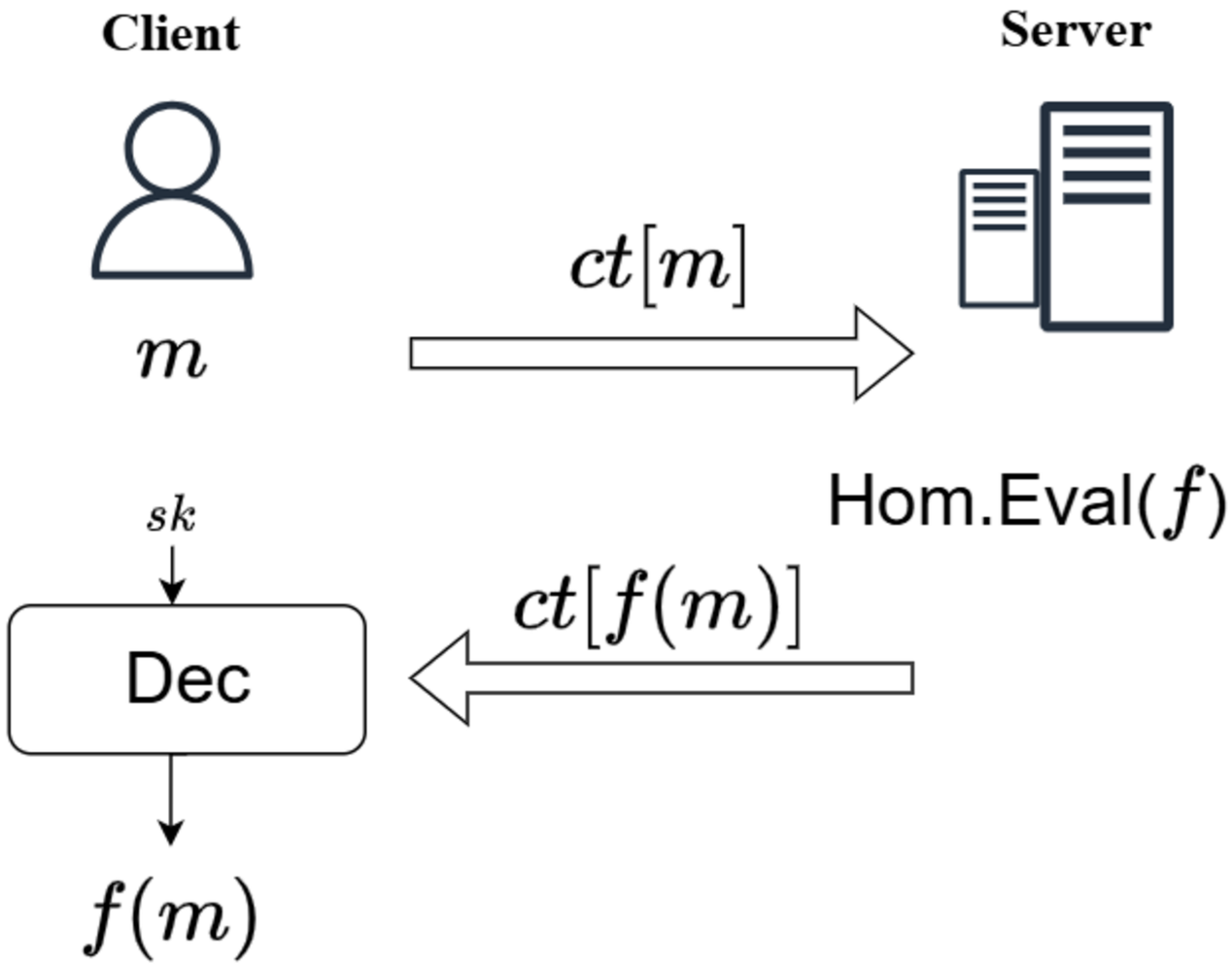

How to achieve verifiable COED (vCOED)?

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Two vCOED approaches

vFHE

- [VKH23, GNS23, ABP+24, TW24, CCC+25, LLZ+25, WWX+25, ...]
- prove **ciphertext** relations

$$\mathrm{Prove}_{\mathrm{Hom.Eval}(\mathrm{F})}$$

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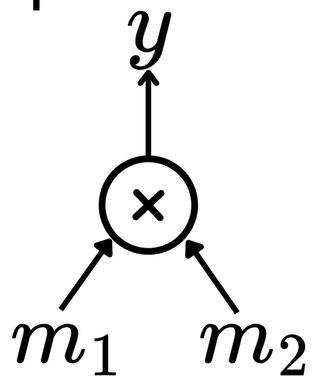

Prove

$$ct[y] = \text{Hom. Mult}(ct[m_1], ct[m_2])$$

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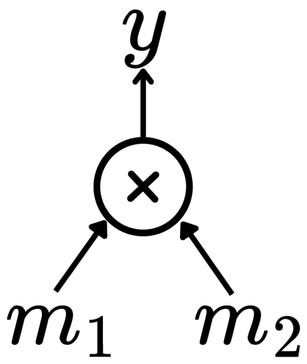

- $$[0 \quad 1 \quad 0 \quad 0] \begin{bmatrix} 1 \\ m_1 \\ m_2 \\ y \end{bmatrix} \circ [0 \quad 0 \quad 1 \quad 0] \begin{bmatrix} 1 \\ m_1 \\ m_2 \\ y \end{bmatrix} = [0 \quad 0 \quad 0 \quad 1] \begin{bmatrix} 1 \\ m_1 \\ m_2 \\ y \end{bmatrix} \quad (1)$$

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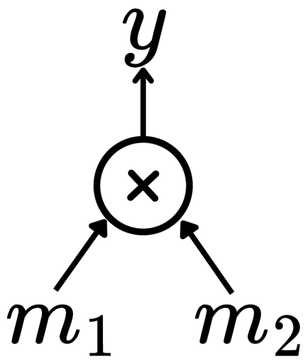

zkSNARK proves the following R1CS relation

- $$\begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\ m_1\\ m_2\\ y\end{bmatrix} \circ \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\ m_1\\ m_2\\ y\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\ m_1\\ m_2\\ y\end{bmatrix} \quad (1)$$
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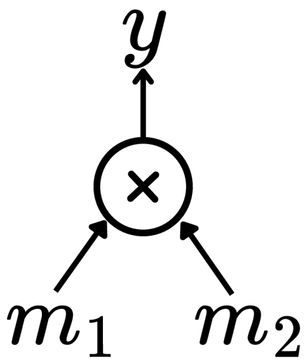

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Blind zkSNARK proves the underlying plaintext relation from ciphertexts

- Given $ct[m_1]$, $ct[m_2]$ and $ct[y]$,

  perform $\mathrm{Hom.Eval}(\mathrm{Prove}_{(1)})$

Warmup Protocol for R1CS [BCR+19]

Given public (sparse) matrices $A, B, C \in \mathbb{F}^{N \times N}$,

the prover wants to prove the knowledge of $z, a, b, c \in \mathbb{F}^{N}$ such that:

$$Az = a$$

$$Bz = b$$

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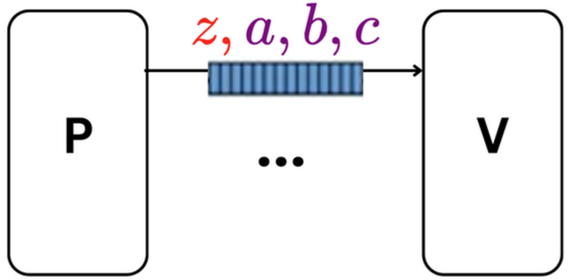

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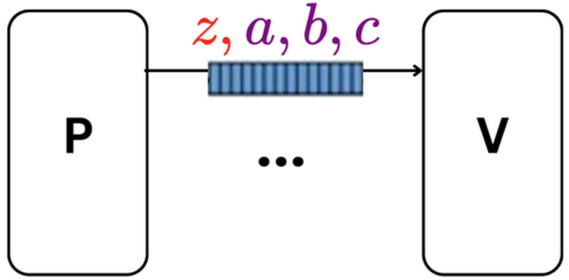

- To obtain sublinear query complexity, Aurora/Fractal uses Reed-Solomon codes

e.g. $z \longrightarrow f_z \in \mathsf{RS}[L, \rho]$

$\| \qquad\qquad \|$

$\hat{f}_z|_H \qquad \hat{f}_z|_L$

Prove R1CS with Aurora/Fractal [BCR+19, COS20]

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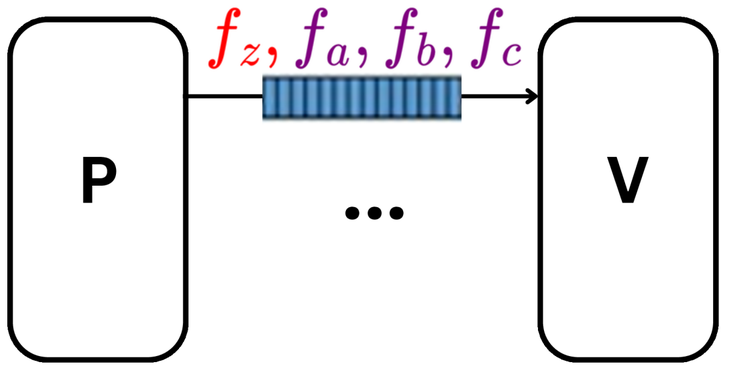

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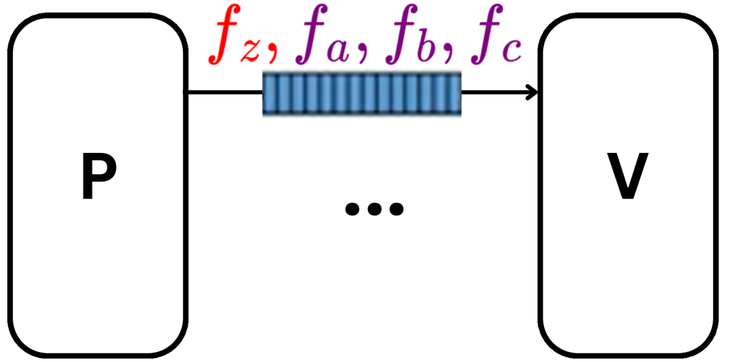

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Low-degree test of the rational constraint

$$Q = \frac{\hat{f}_a \cdot \hat{f}_b - \hat{f}_c}{Z_H}$$

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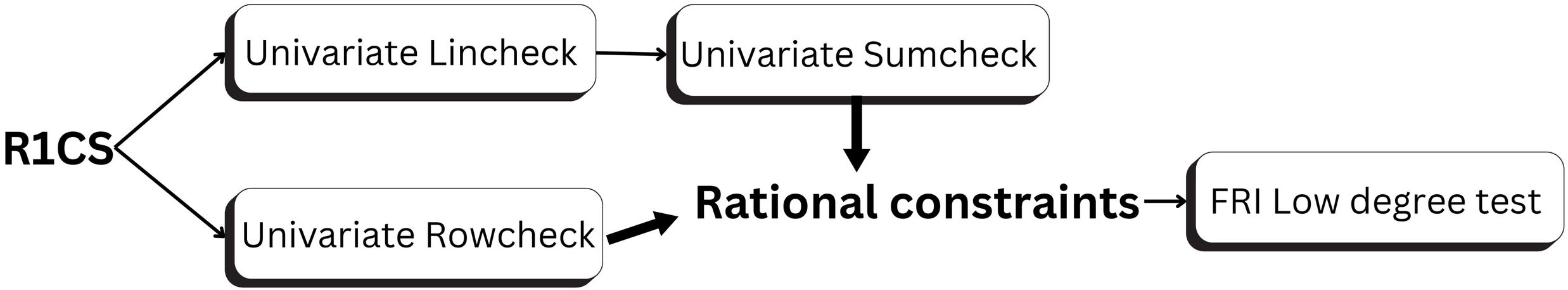

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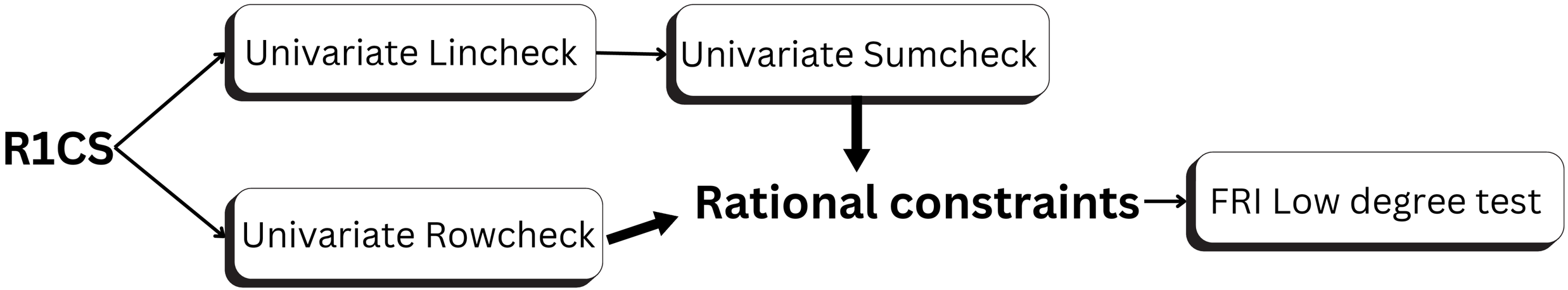

- Blind FRI has been shown to be practical in [ACG+24].
- Our work further demonstrates the practicality of the whole blind Fractal.

Our instantiation of blind zkSNARKs: GBFV for the large-field arithmetic

- We use the GBFV scheme [GV24] to enable native arithmetics of a large field (without emulation)
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Our parameter

	GBFV	BFV
(n,q)	**(12288, 382 bits)**	
slot	$\mathbb{F}_{p^2}$, $p = 2^{64} - 2^{32} + 1$	
#slots	**384**	6144
(Nptct, Nctct)	**(9, 13) bits**	(69, 73) bits

Our instantiation of blind zkSNARKs: Efficient packing-friendly 2D-NTT

- NTT is an important component in Fractal-style zkSNARKs
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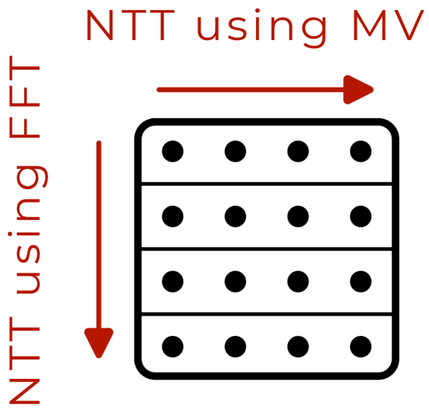
- Our solution: 2D-NTT [CG99]



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  - …
- microbenchmarking for blind Fractal with 2^{20} constraints
  - operation counts and noise estimation are publicly available
  - time estimation: 9.26 hours sequentially

    < 20min assuming 32x speedup from parallelization

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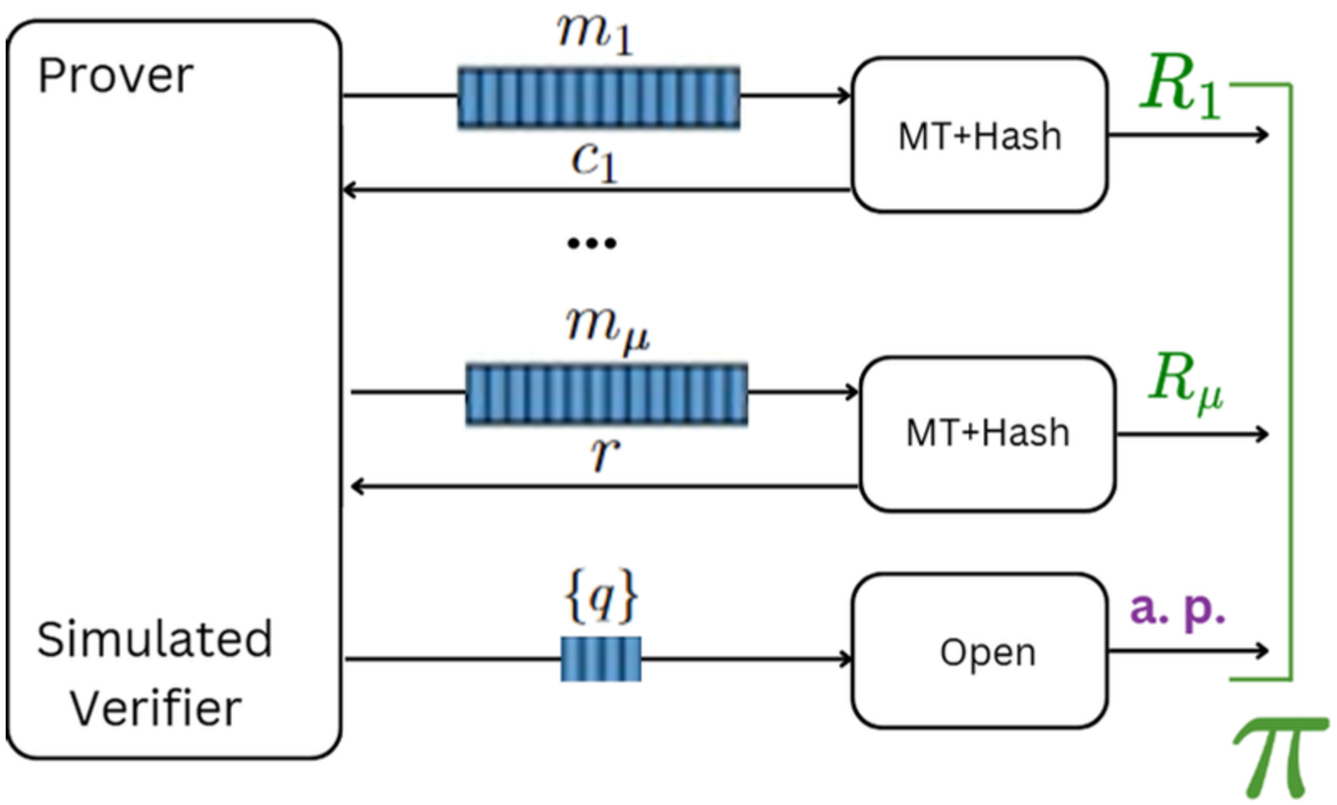
What should the client do for verification?

Non-interactive zero-knowledge proof [BCS16]



Anyone can Verify:

- $f(x, \{q\}, \tau) = 0$
- other consistency checks e.g. the derivation of τ

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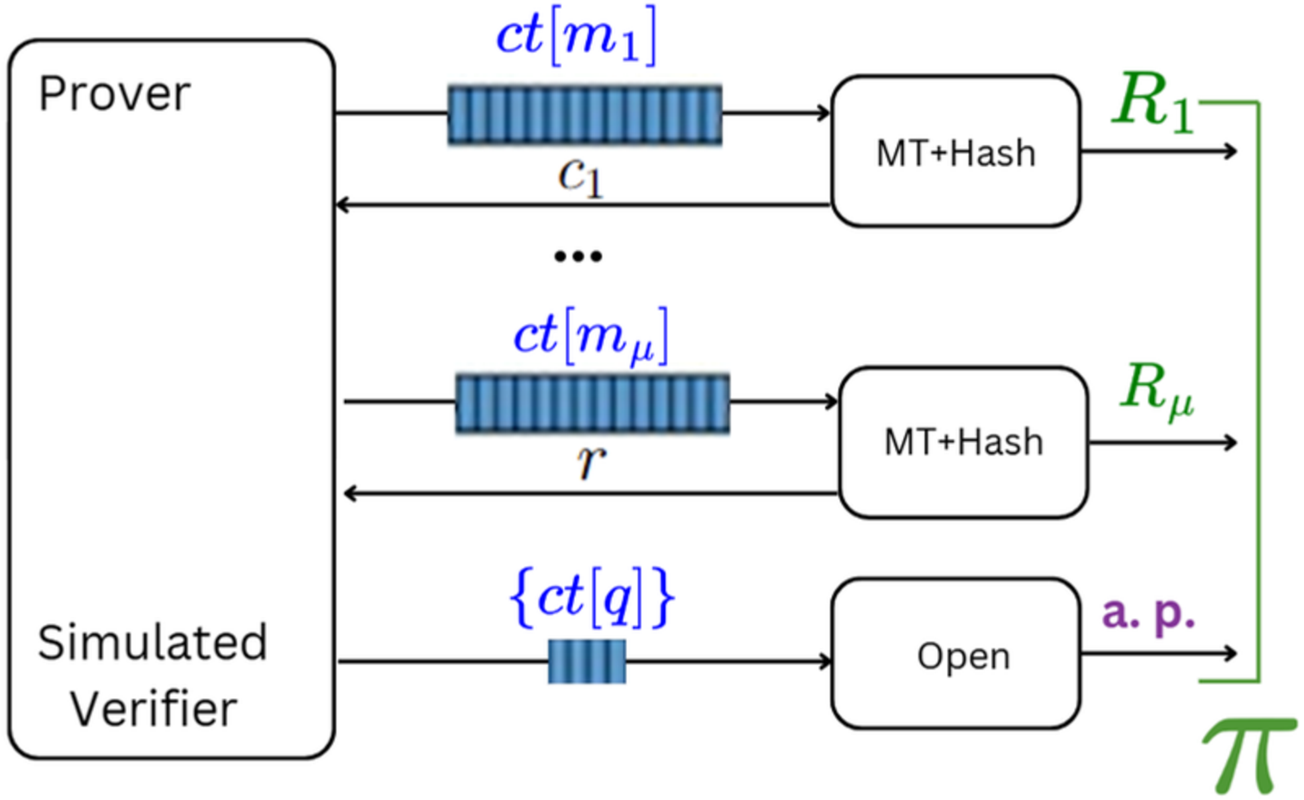

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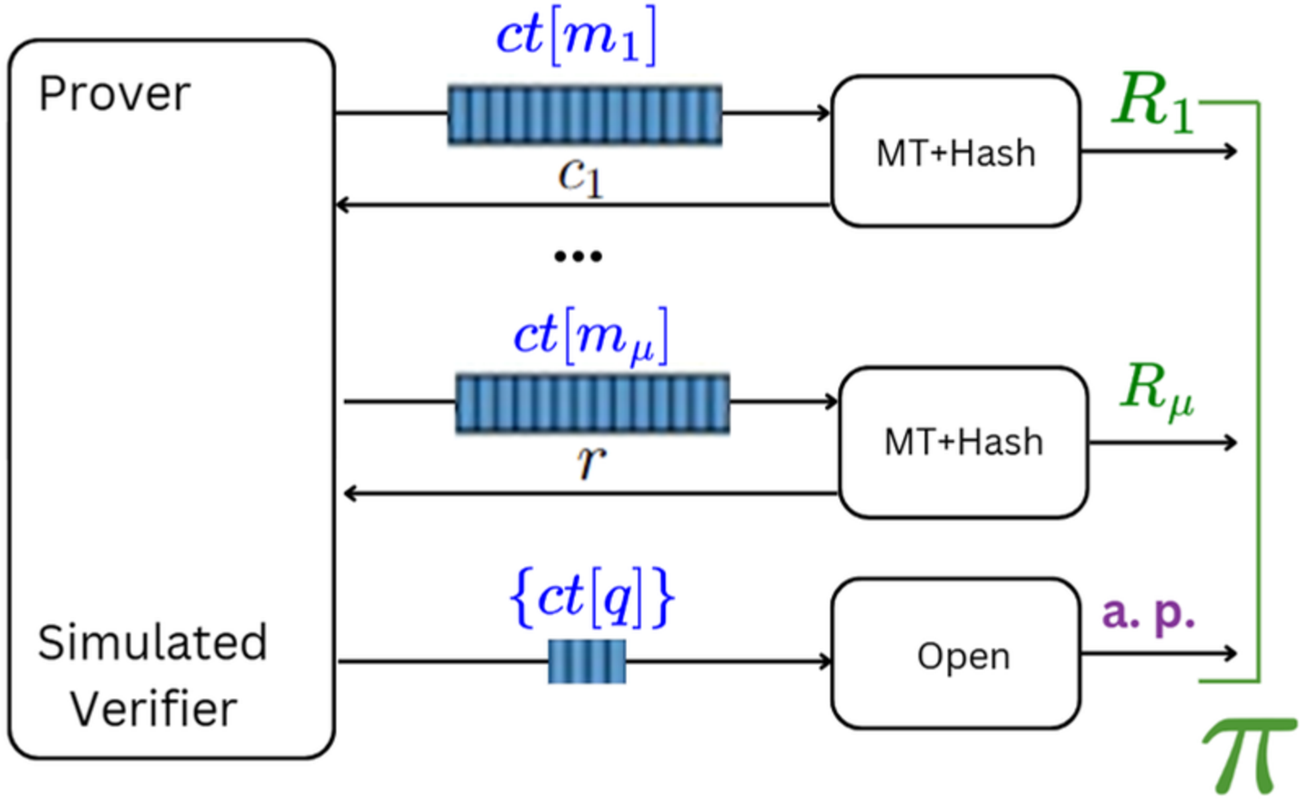

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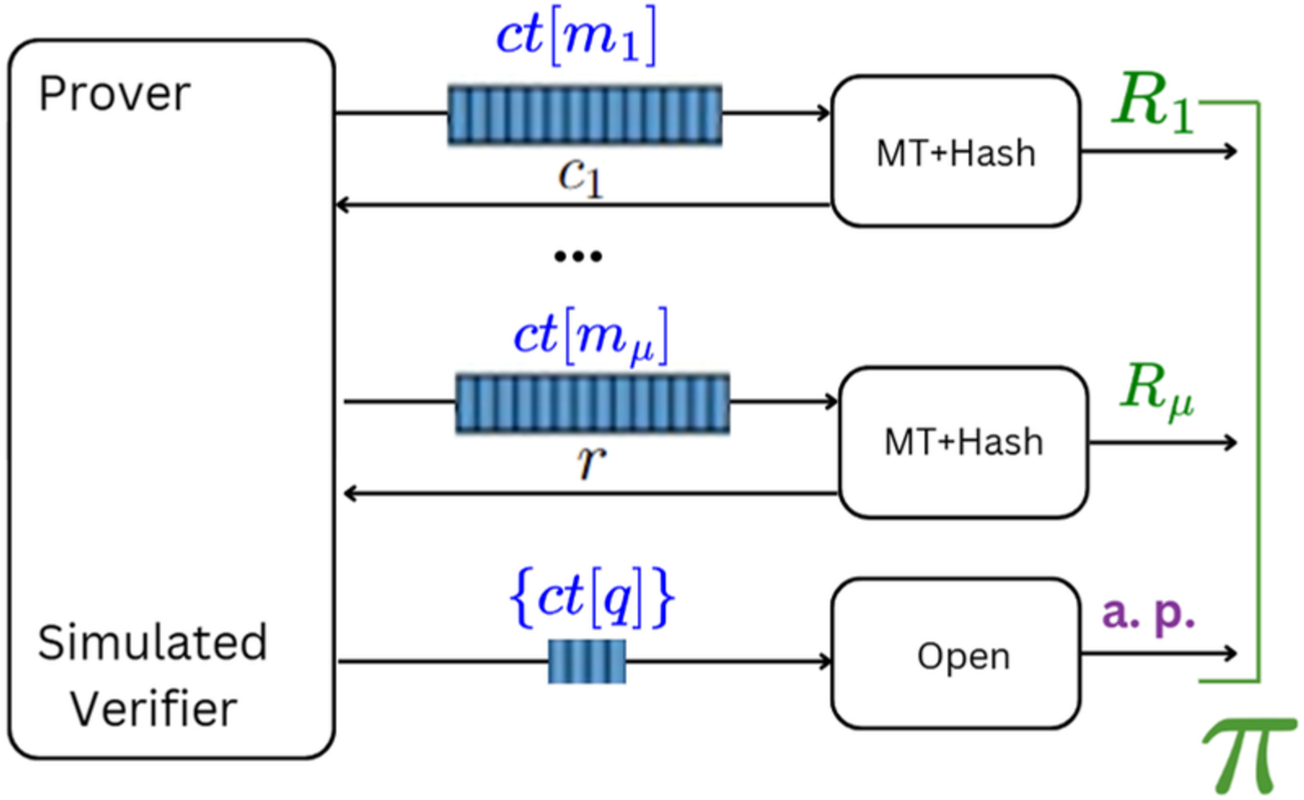

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Goal: to compute F, and convince the public audience of the correct computation

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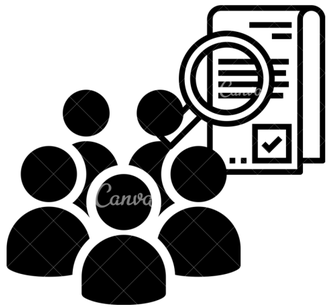

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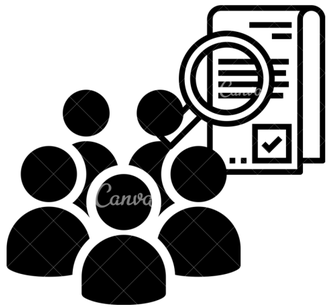

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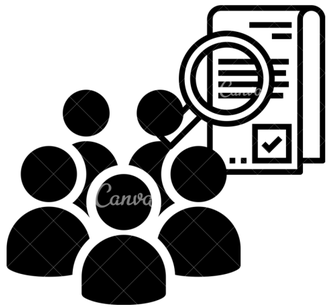

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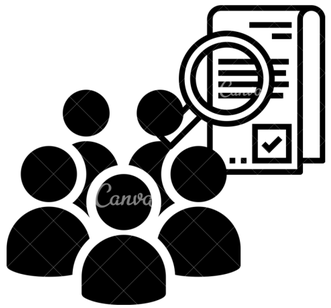

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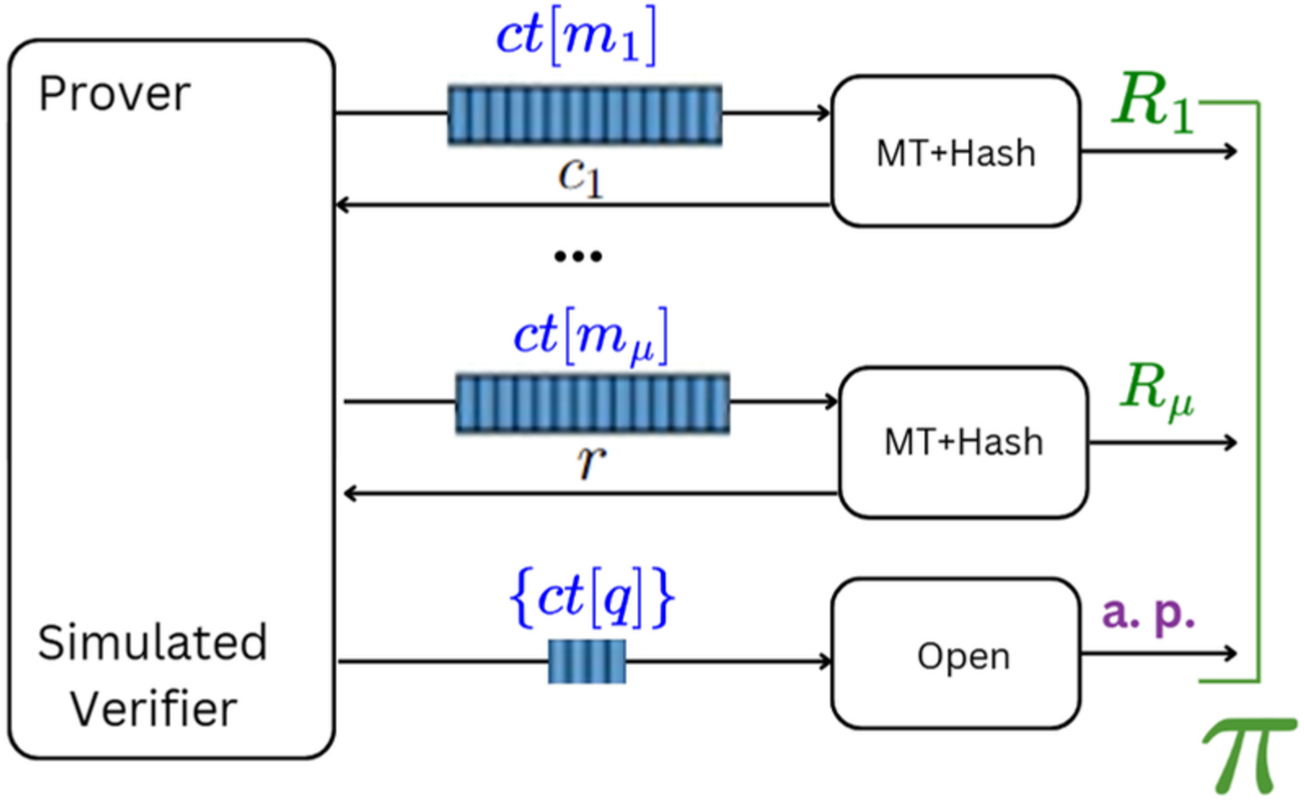

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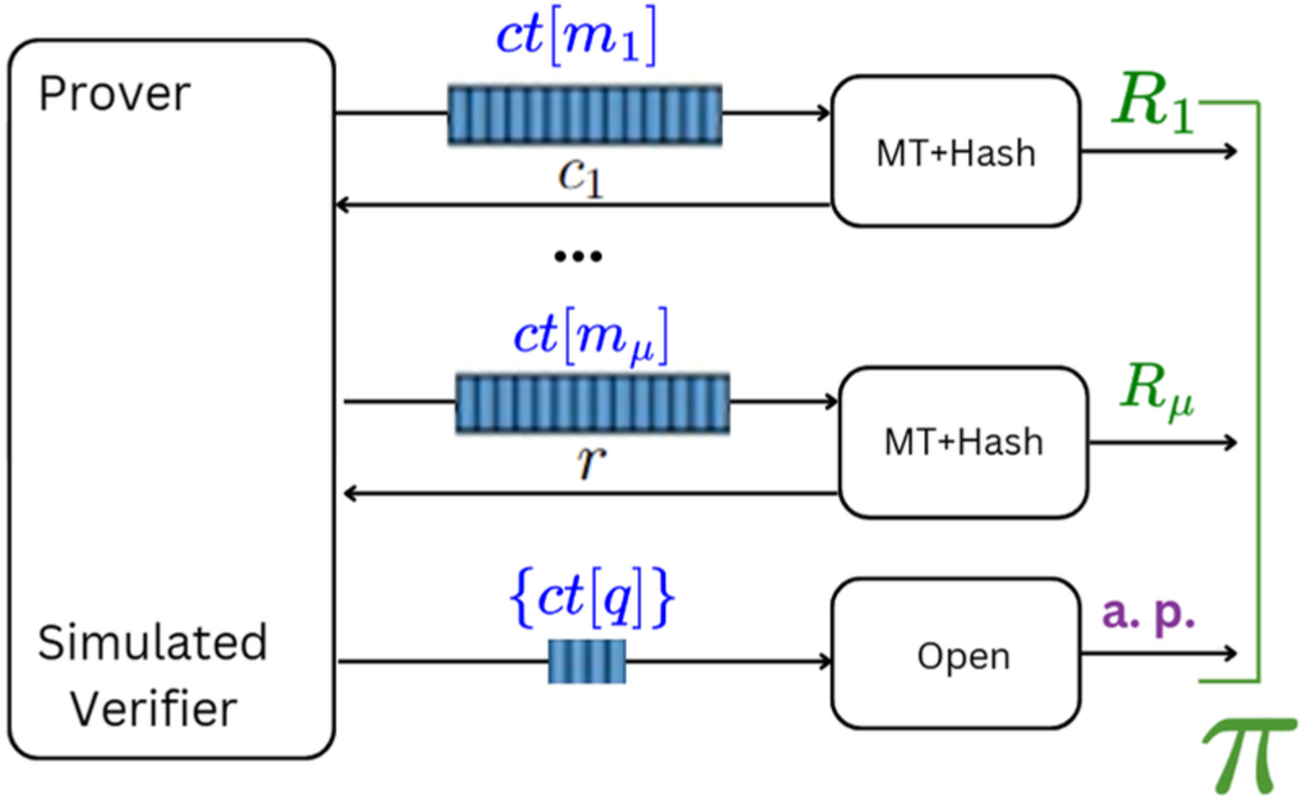

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Make proof publicly verifiable

- append plaintext $\{q\}$
- append Proof of decryption (PoD) for $\{(q, ct[q])\}$

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- Our parameter: $(n, q) = (1536, 48 \text{ bits})$

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- Let $R_{q,n}$ denote the FHE ring with ct modulus q and degree n

$$c_0 + c_1 \cdot s = \Delta \cdot m + e \in R_{q,n}$$

- PoD $\iff$ the noise $e \in R_{q,n}$ is bounded
- Our parameter: $(n, q) = (1536, 48 \text{ bits})$

[LNP22] proof system

- Commitment ring is $R_{q',n'}$ where
  - n' is a power-of-two (typically 64 or 128)
  - q' is chosen s.t. $X^{n'} + 1$ has two irred. fact. mod q'

Proof of decryption: our techniques

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Performance

- Implementation of vec-LNP is publicly available
- ~2 seconds to prove the decryption of ~2500 cts (single thread)

vFHE

- [VKH23, GNS23, ABP+24, TW24, CCC+25, LLZ+25, WWX+25, ...]
- prove **ciphertext** relations

$$\mathrm{Prove}_{\mathrm{Hom.Eval(F)}}$$

- ~ 20 mins to prove $< 2^{10}$ R1CS plaintext constraints
- can reuse sk in delegation

Blind zkSNARK

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Thanks for listening!

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